CALCULATION OF PLANE ELECTRIC FIELDS IN CHANNELS OF MAGNETOHYDRODYNAMIC INSTRUMENTS

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UDC 538.4

The local and integral characteristics of flat MHD channels are studied with allowance for longitudinal and transverse edge effects and heterogeneities in the distributions of conductivity and stream velocity. An analysis is made of the effect of the finite dimensions of the insulating inserts in the longitudinal edge effect and of the modular construction of the side wall in the transverse edge effect on the output parameters of MHD channels. The solution of the problem is based on reduction of the initial quasilinear elliptical equation for the electrical potential with allowance for Ohm's law to an integral equation of the Fredholm type relative to the current density.

1. Introduction. The output parameters of conduction magnetohydrodynamic (MHD) instruments depend strongly on the nature of the distribution of velocity **v**, conductivity σ, and external magnetic field **B**, as well as on the construction of the conducting and insulating walls of the MHD channel. The development of this dependence is connected with the calculation of the electric field in the channel for a fixed wall geometry and given distributions of **v**, σ, and B (kinematic problems). Such problems usually are solved in a two-dimensional formulation which in many cases is allowed by the actual three-dimensional nature of the electric field in the channel.

The construction of solutions of two-dimensional kinematic problems plays an important role in the study of flows with allowance for the MHD interaction, which can be made on the basis of a modular solution of the gasdynamical and electrodynamical problems using iteration algorithms [1].

The main types of problems considered are connected with the study of the transverse and longitudinal edge effects in MHD channels. A detailed theoretical analysis of these effects with a survey of the literature is given in the monographs [2, 3], as well as in [4-7]. These problems usually come down to the solution of an elliptical equation for the electrical potential φ [2]. Such solutions can be constructed in finite analytical form only for particular cases, for example, when the equation for the potential reduces to the Laplace or Poisson equations. This is possible if the magnetic Reynolds number R_m is small and the conductivity of the medium is constant. The second condition is not satisfied for many problems of practical importance connected with the motion of a conducting gas.

When the distribution of conductivity and velocity is nonuniform the analysis of MHD flow becomes complicated. In general it can be based on the use of direct difference methods [6], although in problems with discontinuous boundary conditions the use of these methods requires a large number of calculation points.

Effective algorithms for the calculation of electric fields in the channels of MHD instruments with allowance for sectioning of the walls and nonuniform distributions of velocity, conductivity of the stream, and magnetic field can also be constructed on the basis of integral representations. It should be noted that the kernels of the integral equations can to a certain extent "absorb" the discontinuity of the solutions at the boundaries between insulators and conductors, which permits a considerable reduction in the number of calculation points compared with direct numerical methods. In addition, in a number of cases the conver-

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 8-16, March-April, 1974. Original article submitted August 1, 1973.

sion to integral representations facilitates the proof of uniqueness of the solutions obtained, which often represents an independent problem in finite-difference methods. Integral representations in the solution of electrodynamic problems prove to be quite effective in the calculation of electric and magnetic fields (see [7-10], for example).

2. Integral Equations for the Electrical Potential and Current Density. Let us consider the two-dimensional MHD flow of a conducting medium with a velocity $\mathbf{v}$ (x, y) and conductivity σ(x, y) in an external magnetic field $\mathbf{B}$ (x, y), (div $\mathbf{B}$ =0, rot $\mathbf{B}$ =0).

At small magnetic Reynolds numbers R_m and Hall parameter β the distributions of current density j and potential φ in dimensionless form are described by the equations

$$\mathbf{j} = \sigma (\mathbf{v} \times \mathbf{B} - \nabla\varphi) \tag{2.1}$$

$$\Delta\varphi = (\mathbf{v} \times \mathbf{B} - \nabla\varphi)\, \nabla \ln \sigma + \mathbf{B} \operatorname{rot} \mathbf{v} \tag{2.2}$$

with the boundary conditions at the electrodes

$$\varphi = \pm \varphi_e \tag{2.3}$$

at the grounded conducting parts of the construction

$$\varphi = 0 \tag{2.4}$$

and at the insulators

$$\partial\varphi / \partial n = (\mathbf{v} \times \mathbf{B})\mathbf{n} \tag{2.5}$$

where $\mathbf{n}$ is the unit normal to the channel surface.

Here the dimensions are with respect to h, $\mathbf{v}$ is with respect to v_0, $\mathbf{B}$ to B_0, σ to σ_0, j to $\sigma_0 v_0 B_0$, and φ to v_0B_0h; h and v_0 are half the interelectrode distance and the velocity at the entrance to the MHD channel averaged over the cross section, respectively; B_0 is the maximum induction of the external magnetic field in the channel. The following value is taken as the characteristic conductivity:

$$\sigma_0 = h \Big/ \int_0^h \frac{dy}{\sigma} \quad \text{at} \quad x = 0$$

The main difficulty in the application of integral methods to the solution of equations of the type of (2.2) is connected with the construction of the influence functions determining the structure of the kernels of the corresponding integral equations. A search for such functions directly for (2.2) is possible only in individual cases (for example, when (2.2) is reduced to an equation of the type

$\Delta\varphi + k\varphi = f(x, y)$, where k =const [7, 8]).

Broader possibilities are connected with integral representations of general elliptical equations through the Green function for the Laplace equation (see [11], for example). Such a function is determined only by the boundary conditions of the problem and does not depend on the distribution of conductivity, the stream velocity, or the topography of the magnetic field. The Green function can be constructed for a wide class of problems with the help of a conformal transformation of the regions being considered onto canonical regions in which Green's function is known.

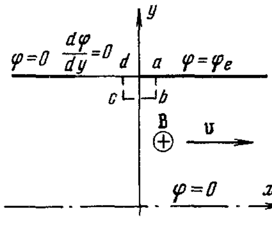
The Green function of the mixed boundary problem for the Laplace equation $\Delta\varphi$ =0 has the form [12]

$$G(x_0, y_0, x, y) = -\ln \sqrt{(x - x_0)^2 + (y - y_0)^2} + g(x_0, y_0, x, y)$$

where g (x_0, y_0, x, y) is an analytical function which assures that the following boundary conditions for G, corresponding to the conditions (2.3)-(2.5), are satisfied: G=0 at the surface of the metallic walls, dG/dn = 0 at the insulators.

Applying the second Green equation to (2.2) and using the boundary conditions for φ and G, we obtain

$$\begin{aligned}\Omega\varphi(x_0, y_0) = &\int_{\Gamma_e} \varphi(\gamma) \frac{\partial G(x_0, y_0, \gamma)}{\partial n} d\gamma - \int_{\Gamma_i} [\mathbf{v}(\gamma) \times \mathbf{B}(\gamma)]\, \mathbf{n} G(x_0, y_0, \gamma)\, d\gamma \\ &+ \iint_S \{[\mathbf{v}(x, y) \times \mathbf{B}(x, y) - \nabla\varphi(x, y)]\, \nabla \ln \sigma(x, y) + \mathbf{B}(x, y) \operatorname{rot} \mathbf{v}(x, y)\} G(x_0, y_0, x, y)\, dx\, dy\end{aligned} \tag{2.6}$$



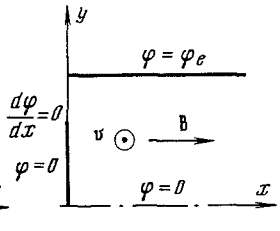


Fig. 1

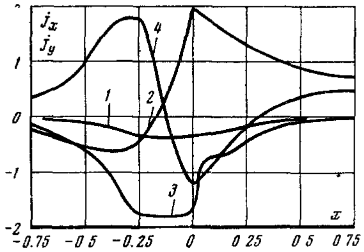


Fig. 2

where Γ_e is the part of the boundary of the region corresponding to the conducting walls while Γ_i is the s same for the insulating walls, and S is the region of the channel

$$\Omega = \begin{cases} -2\pi, & (x_0, y_0) \in S \\ -\pi, & (x_0, y_0) \in \Gamma \\ 0, & (x_0, y_0) \notin S \cup \Gamma \end{cases}$$

Equation (2.6) can be reduced to an integral equation relative to φ by integration by parts using (2.1).

Because of the discontinuous boundary conditions the gradient $\nabla\varphi$ can have a singularity whose nature is not known beforehand. The results of the analysis will be correct in the strict sense if the product $\nabla\varphi\ (\nabla \ln \sigma)\ G$ is integrable, which is satisfied in a majority of practical cases.

The distribution of current density over the channel cross section is of the greatest practical interest. By determining the gradient ∇_0 with respect to x_0 and y_0 from the right and left sides of (2.6) and using (2.1) we obtain an integral equation relative to the current density $\mathbf{j}$ (j_x, j_y)

$$\mathbf{j}(x_0, y_0) = \sigma(x_0, y_0)\Big\{\mathbf{v}(x_0, y_0)\times\mathbf{B}(x_0, y_0) - \frac{1}{\Omega}\Big[\int\limits_{\Gamma_e}\varphi(\gamma)\,\nabla_0\frac{\partial G(x_0, y_0, \gamma)}{\partial n}\,d\gamma - \int\limits_{\Gamma_i}\{[\mathbf{v}(\gamma)\times\mathbf{B}(\gamma)]\mathbf{n}\}$$

$$\times\nabla_0 G(x_0, y_0, \gamma)\,d\gamma + \iint\limits_S\Big[\mathbf{B}(x, y)\operatorname{rot}\mathbf{v}(x, y) - \mathbf{j}(x, y)\nabla\frac{1}{\sigma(x, y)}\Big]\nabla_0 G(x_0, y_0, x, y)\,dx\,dy\Big]\Big\} \qquad (2.7)$$

which is equivalent to a system of two integral equations relative to the corresponding components j_x and j_y.

The integral equation (2.7) has a singularity because of $\nabla_0 G_0$. It can be shown that this singularity is weak and (2.7) is an equation of the Fredholm type.

The integral representation was used to calculate the two-dimensional current distribution in a longitudinal cross section of the end zone of a channel and in a transverse cross section of the working section of the channel.

The system of integral equations for j_x and j_y was solved on an electronic computer by the method of finite sums. The removal of the weak singularity in the kernels of the integral equations was done in accordance with [12]. The system was solved by the iteration method which showed good convergence. For example, a relative discrepancy of less than 10^{-5} for two successive iterations was reached in six to eight runs.

3. Construction of Green's Functions. In studying the edge effects in long and wide MHD channels one can neglect the mutual effect of the entrance and exit or of the opposite side walls. In this case as the canonical region it is convenient to use a half-band with the insulator at the base (Fig. 1, a). For such a region the Green function is constructed from the known Green function of the Dirichlet problem for a band by a mirror reflection of this function relative to the q axis and has the form [10]

$$G(p_0, q_0, p, q) = \frac{1}{2}\ln\frac{[\operatorname{ch}(p-p_0) - \cos(q+q_0)]\,[\operatorname{ch}(p+p_0) - \cos(q+q_0)]}{[\operatorname{ch}(p-p_0) - \cos(q-q_0)]\,[\operatorname{ch}(p+p_0) - \cos(q-q_0)]} \qquad (3.1)$$

At the boundary of the canonical region G satisfies the conditions

$$G(p_0, q_0, p, \pi) = G(p_0, q_0, p, 0) = \partial G(p_0, q_0, 0, q)/\partial p = 0$$

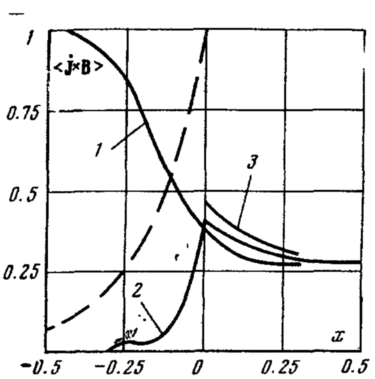


Fig. 3

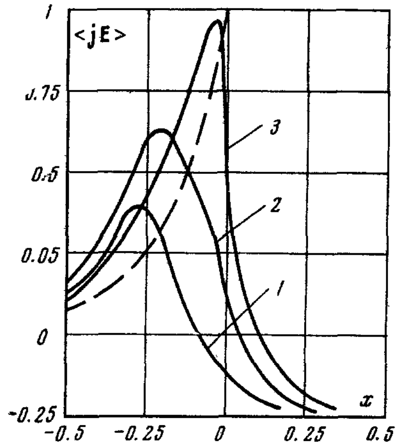


Fig. 4

The Green function (3.1) can be used in the study of end effects in contoured MHD channels when the conducting channels have either nonconducting or metallic walls separated from the electrodes by insulating inserts, as well as in the study of transverse edge effects in channels with conducting modules on the side walls (Fig. 1).

The solution of the problem comes down to a search for the function (3.2)

$$W(x + iy) = p(x, y) + iq(x, y) \tag{3.2}$$

which accomplishes the conformal mapping of the region under consideration onto the half-band.

After determining the mapping function (3.2) one can obtain an analytical expression for the contour integral in (2.7)

$$\int\limits_{\Gamma_e} \varphi(\gamma) \frac{\partial G(x_0, y_0, \gamma)}{\partial n} d\gamma = \Omega \varphi_e q(x_0, y_0) \tag{3.3}$$

which considerably simplifies the calculations in analytical studies.

In numerical calculations of various edge effects in the channels of MHD instruments (see Fig. 1) the transition from one channel studied to another is made by substituting into the calculation program a unit which accomplishes the conformal mapping of the region $\{x, y\}$ being considered onto the canonical region $\{p, q\}$.

4. Longitudinal Edge Effect in a Flat MHD Channel. The distribution of the electric field was studied in a channel whose semiinfinite electrodes are separated from the conducting nozzle by an insulating insert of thickness δ; the half-height of the channel is taken as unity. The dimensionless distributions of velocity $\mathbf{v}\,\{v_x(y), 0, 0\}$ and conductivity $\sigma(y)$ were given in the form

$$v_x = M \frac{\operatorname{ch} M - \operatorname{ch} My}{M \operatorname{ch} M - \operatorname{sh} M}, \quad \sigma = \frac{\sigma_1 + \sigma_2 + 1}{(\sigma_2 + 1)(1 + \sigma_1 y^{\sigma_2})} \tag{4.1}$$

Here M, σ_1, and σ_2 are constants whose variation makes it possible to give different profiles of v_x and σ.

The use of Eqs. (4.1) corresponds to the fact that the velocity and electrical resistance of the working substance averaged over the channel cross section are constant in the direction of the y axis

$$\int\limits_0^1 v_x\, dy = 1, \qquad \int\limits_0^1 \sigma^{-1} dy = 1$$

The external magnetic field was assumed to be a one-component field and to be directed perpendicular to the plane of flow. The dimensionless distribution of the magnetic field at the entrance to the MHD channel was given as uniform in the interelectrode zone (B = 1 for x ≥ 0) and as exponentially attenuated outside it (B = exp (αx) for x < 0). Such an approximation of B at the entrance to an MHD channel has been discussed in detail in [13, 14].

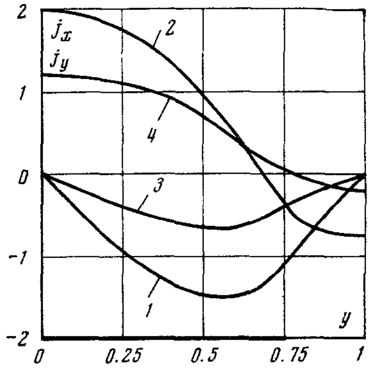


Fig. 5

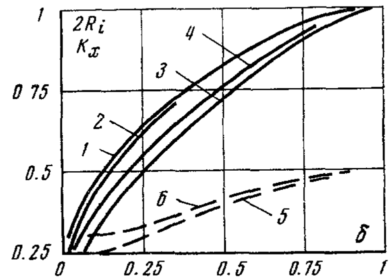


Fig. 6

The mapping of this region onto the half-band was accomplished by the function

$$W(z) = \operatorname{Arch}\left[2\left(1 + e^{\pi z}\right) / \left(1 - e^{-\pi\delta}\right) - 1\right]$$

A squaring trapezoidal equation in the region of $|x| < 3$ and a squaring Hermite equation in the region of $|x| > 3$ were used for the conversion to finite sums in the integral equations. It was assumed that when $|x| > 3$ the distributions $j\,(j_x, j_y)$ are known and coincide with the asymptotic distributions

$$j_x = 0, \quad j_y = 1 - \varphi_e \quad \text{for} \quad x \to \infty$$
$$j_x = 0, \quad j_y = B(x) \quad \text{for} \quad x \to -\infty$$

As the calculations showed, this assumption is realized with an accuracy of $\sim 1\%$.

The calculations were conducted for grids of 50 and 80 nodes and gave practically identical results (the local differences of the two respective solutions did not exceed 1.5%).

In determining the total current of the channel the integration of j_n was carried out not over the edge of the electrode where there are singular points ($j_n \to \infty$) but over a contour near the edge which does not contain points of discontinuity (the contour abcd in Fig. 1a).

The results of numerical calculations of the local and integral characteristics of an MHD channel with $\delta = 0.25$ and $\varphi_e = 0.75$ are presented in Figs. 2-4. The values $M = 1$, $\sigma_1 = 1$, and $\sigma_2 = 3$ were adopted, which corresponds to close to a parabolic velocity profile and to decrease conductivity of the medium in the boundary layer.

The distributions of the j_x and j_y components of the current density along the length of the channel for different values of y are presented in Fig. 2. The dependence of B on x is plotted with a dashed line in Figs. 3 and 4. Curves 1 and 3 of Fig. 2 pertain to the distributions of $j_x(x)$ and curves 2 and 4 to the distributions of $j_y(x)$. Curves 1 and 2 correspond to $y = 0.05$ and curves 3 and 4 to $y = 0.95$.

It is seen from the nature of the variation in the components of the current density near the channel axis (curves 1 and 2) and in the boundary-zone (curves 3 and 4) that with the drop in the magnetic field outside the electrodes the end effect is manifested in the form of leak currents to the metallic parts of the nozzle and an eddy current at the entrance to the MHD channel. As the calculations showed, the size of the insulating insert and the magnetic field gradient exert the main effect on the end losses. A decrease in the conductivity in the boundary regions leads to an increase in the intensity of the eddy current and a decrease in the leak currents along the insulating gap. For a given flow rate in the channel the effect of the velocity profile on the intensity of the eddy current and the nature of the distribution of current density in the MHD channel is relatively small.

The electromagnetic characteristics of the MHD channel averaged over the cross section are of interest for calculations of the integral characteristics of MHD channels and the construction of canonical flows.

Distributions over the channel length of values, averaged over a cross section x=const, of the retarding force $\langle \mathbf{j} \times \mathbf{B} \rangle$ and the electrical power $\langle \mathbf{jE} \rangle$, relative to $\sigma_0 v_0 B_0^2$ and $\sigma_0 v_0^2 B_0^2$, respectively, are presented in Figs. 3 and 4 for different functions of the drop in B. (Curves 1 correspond to $\alpha = 0$, curves 2 to $\alpha = 5$, curves 3 to $\alpha = \infty$; the dashed curve shows the variation in B for $\alpha = 5$.)

It is seen that the influence of the edge effect on the characteristics of the MHD channel averaged over the cross section is exerted at a distance of ~ 0.25 diameters from the edge of the electrode. In the rest of the electrode region the values of $\langle \mathbf{j} \times \mathbf{B} \rangle$ and $\langle \mathbf{j}\mathbf{E} \rangle$ are close to the asymptotic values. A reduction in the conductivity in the boundary layer leads to an increase in the averaged retarding force near the electrode edges of the channel. This is explained by the decrease in the leak currents and the increase in the intensity of the eddy current. With a sharp drop in the magnetic field outside the electrodes ($\alpha \to \infty$) the retardation of the flow in the electrode zone is intensified (curve 3 in Fig. 3).

The extension of the field outside the limits of the electrodes decreases the retarding force in the electrode zone at the entrance to the MHD channel. In this case the retarding force increases sharply due to the short-circuiting currents at the conducting walls of the nozzle, which leads to a general increase in the retardation of the flow at the entrance to the MHD channel.

Besides the retardation of the flow at the entrance, the presence of leak currents to the grounded elements of the construction and the eddy current cause a considerable redistribution of energy in the entrance zone of the MHD channel (see curve 1 in Fig. 4). The supplying of energy to the stream is observed in the zone of the nozzle and the insulating gap.

The calculations show that the amount of energy supplied to the stream increases considerably with an increase in the steepness of the drop in the field outside the electrode zone. This is explained by the growth in the intensity of the eddy current at the entrance to the MHD channel. With an increase in α the maximum in the energy supplied shifts to the electrode. In the initial section of the electrode zone of the channel the generated power averaged over the cross section can exceed the corresponding value in the central zone.

5. Transverse Edge Effect in a Rectangular MHD Channel. The distribution of the electric field was studied in a transverse cross section of a rectangular MHD channel with a modular side wall (Fig. 1, b). The half-height of the channel is taken as equal to unity and the width of the insulating insert is δ. The external magnetic field was assumed to be uniform and parallel to the electrodes, and the distributions of velocity and conductivity in the interelectrode gap were given as in Part 4. It is shown in [4] that in rectangular MHD channels with a relative electrode width $L \geq 2$ the mutual influence of the edge effects at the side walls can be neglected and a semiinfinite channel can be considered.

The mapping of the initial semiinfinite region onto the canonical region was accomplished by the function

$$W(z) = \operatorname{Arch}\left[2(\cos \pi z + 1)/(1 - \cos \pi\delta) - 1\right]$$

The distributions of the j_x and j_y components of the current density over the height of the channel for different values of x with $\delta = 0.25$, $\varphi_e = 0.75$, $M = 1$, $\sigma_1 = 1$, and $\sigma_2 = 3$ are presented in Fig. 5. Curves 1 and 3 pertain to the distributions of $j_x(y)$ and curves 2 and 4 to the distributions of $j_y(y)$. Curves 1 and 2 correspond to $x = 0.25$ and curves 3 and 4 to $x = 0.75$.

The distributions of current density in the direction of the y axis near the side wall (curves 1 and 2) indicate the presence of an intense connected eddy current at the conducting module. A comparison of the data obtained with the results of a study of the transverse edge effect with $\sigma = \text{const}$ and $v = \text{const}$ made in [4] showed that with the means adopted for making σ dimensionless (4.1) the reduction in σ in the electrode zone leads to a decrease in the leak currents and an increase in the intensity of the connected eddy current.

The coefficient of reduction in the no-load voltage $k_x = \varphi_{exx}/v_0 B_0 h$ and the internal resistance of the MHD channel R_i as functions of the relative fraction of the insulating gap δ for different profiles of v_x and σ are given in Fig. 6. In determining k_x and R_i it was assumed that the electrode width is equal to two. The solid lines pertain to the dependences of k_x on δ and the dashed lines to the dependences of R_i on δ. Curve 1 corresponds to the calculation with $\sigma = \text{const}$ and $M = \infty$ ($v_x = \text{const}$); 2) $\sigma = \text{const}$, $M = 10$ (turbulent profile); 3) $\sigma = \text{const}$, $M = 0$ (Poiseuille profile); 4) $\sigma = \text{var}$ ($\sigma_1 = 1$, $\sigma_2 = 3$), $M = 0$; 5, 6) calculated dependences of R_i on δ for $\sigma = \text{const}$ and $\sigma = \text{var}$ ($\sigma_1 = 1$, $\sigma_2 = 3$). Because of the quasilinear nature of the problem, the internal resistance of the MHD channel does not depend on the velocity distribution and is determined by the channel geometry and the profile of the conductivity σ (x, y) [2]. It is seen that for turbulent flow of the medium with constant conductivity the coefficient of reduction in the no-load voltage practically coincides with k_x for flow with $v_x = \text{const}$ (the difference is 1-2%). The latter makes it possible to calculate the no-load voltage of an MHD channel with turbulent flow of the working substance from the analytical dependences of [4].

For Poiseuille flow (curve 3) the difference of k_x from the no-load voltage of the MHD channel with v_x = const is significant and reaches 15% for δ =0.25. Allowance for the heterogeneity in the distribution of σ (curve 4) leads to an increase in k_x compared with curve 3 (σ =const and M=0). This is explained by the decrease in leak currents along the insulating gap with a reduction in σ in the electrode zone.

Curves 5 and 6 in Fig. 6 show that with a decrease in the relative fraction of the insulating gap at the side wall the internal resistance of the MHD channel decreases, which is explained by the increase in the shunting effect of the conducting module. A comparison of the curves with σ =const and σ =var shows the weak dependence of the total internal resistance of the MHD channel on the profile of the conductivity σ when the condition $\int_0^1 \sigma^{-1} dy = 1$ is satisfied.

The examples considered show the possibility of applying integral methods to the calculation of complicated electric fields described by anharmonic potentials by using the apparatus of Green functions of the Laplace equation. The integral method can be applied to the study of edge effects in various MHD channels with curvilinear walls when the region of the channel admits the conformal mapping onto a canonical half-band. The results obtained can be considered as strict in the sense that thanks to the uniqueness of the solutions of integral Fredholm equations they can always be sufficiently approximated to exact solutions with the help of a computer. The development of the method is postulated on the possibility in principle of its use for problems with $R_m \geq 1$ and $\beta \geq 1$ and with electrodes of finite dimensions. In the latter case two Green functions are constructed for the corresponding boundary zones with subsequent matching of the solutions.

In conclusion the authors thank L. A. Vulis,* A. V. Gubarev, and A. L. Genkin for discussion of the formulation of the problem and the results of the work.

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